Collective modes in color superconductors

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We investigate the fluctuation effects of a gap parameter in color superconductors. The fluctuation modes are described as the collective motion of a diquark field. One of the modes is a massless Nambu-Goldstone boson and the other is a diquark boson whose mass is twice the gap energy. It is also shown that in the normal phase the fluctuation mode becomes a precursory (soft) mode whose mass and decay width are calculated near the critical temperature. We discuss the temperature dependence of such quantities in detail.

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I. INTRODUCTION

In many-body systems, spontaneous symmetry breaking [1] is a key concept in understanding the structure of the vacuum state. For example, the gauge invariance is broken in the ground state of the superconductors and is restored at higher temperature than the critical point, T_c. The matter is in the Nambu-Goldstone (NG) phase ($T<T_c$) where the symmetry is spontaneously broken and in the Wigner phase ($T>T_c$) where it is not broken. There is an order parameter that discriminates between two such phases; its value is positive (nonzero) in the NG phase and zero in the Wigner one. Such an order parameter is nothing but the gap energy in the superconductors. It is connected to a mean field in the framework of the Hartree-Fock (Bogoliubov) approximation.

Usually the order parameter is fluctuating around its mean field. This fluctuation is also seen in the Wigner phase and plays an important role near the critical temperature. The large fluctuation in the Wigner phase is a precursory mode of the spontaneous symmetry breaking and is called a soft mode. There are several examples of such soft modes, such as the paramagnon in the ferromagnet, the pairing vibration in spherical nuclei, and the paracurrent in the superconductors. In hadron physics, Hatsuda and Kunihiro [2–4] found a similar phenomenon concerning the chiral symmetry. The order parameter of this symmetry is $\langle q\bar{q}\rangle$ where q means a quark-antiquark field and its fluctuation modes in the NG phase are pions and σ mesons. They discuss the precursor of this symmetry in [4].

Recently another symmetry breaking has been introduced in hadron physics; the order parameter is $\langle qq\rangle$ and is called color-superconductivity [5–10]. It is the purpose of this paper to discuss the soft modes concerning this order parameter. Generally there are two kinds of fluctuation modes. One is the phase mode which is called the Nambu-Goldstone one and the other is the amplitude mode. The former has been studied by many authors recently [11]. Our main interest is to study the latter mode, particularly to calculate the mass and the decay width of the collective modes. Their temperature dependence is also investigated near the critical temperature. It is also pointed out that this mode is a soft mode that is known as a precursor. Since such a mode is observed in the spectral function, we will also calculate the spectral function in our formalism.

In the next section, we derive an effective action for the order parameter, pair field of the quark matter at finite density and/or temperature. In Sec. III, a mean field (BCS) approximation is carried out and then fluctuation around it is considered. The fluctuation (collective) modes are described by scalar fields. The mass and decay width of the modes are calculated numerically in Sec. IV. Finally some comments are added.

II. EFFECTIVE ACTION

We consider the quark matter with three flavors. For concreteness we will take the following four-Fermi interaction:

$$\mathcal{L}=\bar{\psi}(i\gamma\partial+\mu\gamma^0)\psi+g\sum_a(\bar{\psi}\gamma_\mu\lambda^a\psi)(\bar{\psi}\gamma^\mu\lambda^a\psi), \quad (1)$$

where g, μ, and λ^a denote the coupling constant, chemical potential, and flavor SU(3) matrix, respectively. This interaction is the one-gluon exchange type. However, instead of a simple gluon propagator we use an instantaneous contact interaction. This interaction has been used as a model Lagrangian instead of QCD. In order to investigate the pairing correlation, we make use of the Fierz transformation on the right-hand side. Then the Lagrangian is rewritten as

$$\mathcal{L}=\bar{\psi}(i\gamma\partial+\mu\gamma^0)\psi+\tfrac{2}{3}g \times\sum_{a,b=2,5,7}{}'(\bar{\psi}\gamma_5C\lambda^a\Lambda^b\bar{\psi}^t)(\psi^tC^{-1}\gamma^5\lambda^a\Lambda^b\psi). \quad (2)$$

Here we have left only the most attractive terms, which are spin-singlet, color-triplet, and flavor-triplet pairs, from the various terms. This means that the λ_a and Λ_b should be restricted to antisymmetric SU(3) matrices ($a,b=2,5,7$).

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The partition function of our system at finite temperature T is represented in the functional form by

$$Z=\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp(-S), \tag{3}$$

where S is a Euclid action defined by $S=\int_0^\beta d\tau d\vec{r}\mathcal{L}$ ($\beta = 1/T$ and $\tau=it$).

Now let us introduce an auxiliary scalar field $\phi_\rho(x)$ representing a wave function of a pair of quarks. Then the partition function can be rewritten as

$$Z=\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi^*\mathcal{D}\phi \exp(-S'), \tag{4}$$

where the corresponding Lagrangian is given by

$$\mathcal{L}'=\bar{\psi}(i\gamma\partial+\mu\gamma^0)\psi-\frac{2}{3}g\sum_\rho (\bar{\psi}\gamma_5 CT^\rho\bar{\psi}_\phi\rho^t+\text{H.c.}) +|\phi_\rho|^2. \tag{5}$$

The T^ρ denotes λ_a and Λ_b. To calculate this integral, it is convenient to use Fourier transformations for each field,

$$\psi(x)=\sum_{p,s} \psi(p,s)u(\vec{p},s)e^{i(\vec{p}\vec{x}-\omega\tau)}, \tag{6}$$

$$\phi(x)=\sum_q \phi(q)e^{i(\vec{q}\vec{x}-\nu\tau)}, \tag{7}$$

where the Matsubara frequencies are defined by $\omega=(2n+1)\pi T$ and $\nu=2m\pi T$ (n and m are integers). Substituting these equations into Eq. (5), the action is written in the Nambu representation as follows [12]:

$$S'=\frac{\beta}{2}\sum_{p,p',s}\begin{pmatrix}\psi^*(p,s)\\ s\psi(\tilde{p}',s)\end{pmatrix}^t G^{-1}\begin{pmatrix}\psi(p,s)\\ s\psi^*(\tilde{p}',s)\end{pmatrix} -\beta\sum_q |\psi_\rho(q)|^2, \tag{8}$$

where the propagator is defined by

$$G_0^{-1}\equiv\begin{pmatrix} i\omega-\xi_p & \hat{\Delta}(p,\tilde{p}')\\ \hat{\Delta}^*(\tilde{p}',p) & i\omega'+\xi_{p'}\end{pmatrix}. \tag{9}$$

Here the uses are made of $p=(\vec{p},\omega)$, $q=(\vec{q},\nu)$, and $\tilde{p}=(-\vec{p},-\omega)$. The gap parameter on the right-hand side is defined by

$$\hat{\Delta}(p,\tilde{p}')\equiv\sqrt{\frac{8g}{3}}u^\dagger(\vec{p},s)u(\vec{p}',s)T^\rho\phi_\rho(p'-p). \tag{10}$$

The caret means a matrix in the color-flavor space. On carrying out the integration with respect to the quark fields in the partition function (3), we obtain the partition function expressed in terms of scalar (pairing) fields:

$$Z=\int \mathcal{D}\phi^*\mathcal{D}\phi \exp[-S_{\text{eff}}], \tag{11}$$

where the effective action S_{eff} is defined by

$$S_{\text{eff}}=-\text{Tr}\ln(\beta G^{-1})-\beta\sum_q |\phi_\rho(q)|^2. \tag{12}$$

The effective action given by only the pairing field will play a fundamental role in our later discussions.

III. FLUCTUATIONS

To begin with we introduce a mean field approximation for the effective action. The path integral of the effective action is replaced by a mean field, which should be a stationary field in the path integral. We assume the following constant field:

$$\hat{\Delta}_0(p,\tilde{p}')\equiv\delta_{p,p'}\sqrt{\frac{8g}{3}}\phi_0\sum_a{}'\lambda_a\otimes\Lambda_a. \tag{13}$$

This pair has the most symmetric state in color-flavor space and is known as the color flavor locking state [9], and the matrix will be expressed by

$$\hat{\varepsilon}\equiv\sum_a{}'\lambda_a\otimes\Lambda_a. \tag{14}$$

The constant $\phi_0\equiv\phi(0)$ is determined by the stationary condition of the partition function (11), which is written as

$$\frac{8g}{3\beta}\sum_p \text{tr}\frac{\hat{\varepsilon}^2}{\omega_p^2+\hat{E}_p^2}=1, \tag{15}$$

where the "tr" denotes a trace of color-flavor space. This equation is nothing but the gap equation in the BCS theory.

Next let us consider fluctuation around the constant field by the following equation:

$$G^{-1}=G_0^{-1}+\sqrt{\frac{8g}{3}}\begin{pmatrix}0 & \tilde{\phi}(p,\tilde{p}')\hat{\varepsilon}\\ \tilde{\phi}^*(\tilde{p}',p)\hat{\varepsilon} & 0\end{pmatrix} \equiv G_0^{-1}+\delta G^{-1}. \tag{16}$$

The fluctuation is described by the second term on the right-hand side. Of course there are many other fluctuation modes whose expressions are written by $\delta\hat{\Delta}=\Sigma_{a,b}\phi_{ab}\lambda_a\otimes\Lambda_b$ ($a,b=0,1,2,\ldots,8$, $\lambda_0=\Lambda_0=1$). The discussions on the other modes are similar to those in this section. For convenience our discussions are restricted to the fluctuation of Eq. (16).

Substituting this equation into the effective action (12), we get

$$S_{\text{eff}}=S_{\text{eff}}^{(0)}+S_{\text{eff}}^{(1)}+S_{\text{eff}}^{(2)}+\cdots. \tag{17}$$

The first term is nothing but the mean field (BCS) action. The second term should vanish by the stationary condition mentioned above. The third term describes the fluctuation and is given by

$$S_{\rm eff}^{(2)}=\frac{1}{2}{\rm Tr}(G_0\delta G_0^{-1})^2+\beta\sum_q |\tilde{\phi}(q)|^2. \tag{18}$$

The BCS propagator G_0 on the right-hand side is written as

$$G_0=\begin{pmatrix} -i\omega_p-\xi_p & \hat{\Delta}_0(p,\tilde{p}') \\ \hat{\Delta}_0^*(\tilde{p}',p) & -i\omega_{p'}+\xi_{p'} \end{pmatrix}(\omega_p^2+\hat{E}_p^2)^{-1}. \tag{19}$$

The $\hat{E}_p$ denotes a quasiparticle energy and is defined by $\hat{E}_p^2=\xi_p^2+\hat{\Delta}_0^\dagger\hat{\Delta}_0$. Substituting this equation into Eq. (18) and rewriting it in the matrix form, the action of second order is transformed into

$$S_{\rm eff}^{(2)}=\sum_q (\phi^\dagger(q) \quad \phi(-q))\begin{pmatrix} X_q & Y_q \\ Y_q^* & X_q^* \end{pmatrix}\begin{pmatrix} \phi(q) \\ \phi^*(-q) \end{pmatrix}. \tag{20}$$

The matrix elements on the right-hand side are defined by

$$X_q=\frac{4g}{3}\sum_p {\rm tr}\left[\frac{(i\omega_p+\xi_p)(i\omega_{p-q}-\xi_{p-q})\hat{\epsilon}^2}{(\omega_p^2+\hat{E}_p^2)(\omega_{p-q}^2+\hat{E}_{p-q}^2)}\right]\times|N_{p,p-q}|^2+\frac{1}{2}\beta, \tag{21}$$

$$Y_q=\frac{4g}{3}\sum_p {\rm tr}\left[\frac{\hat{\Delta}_0^\dagger\hat{\Delta}_0\hat{\epsilon}^2}{(\omega_p^2+\hat{E}_p^2)(\omega_{p-q}^2+\hat{E}_{p-q}^2)}\right]|N_{p,p-q}|^2. \tag{22}$$

Instead of the complex field $\phi(q)$, let us introduce two real scalar fields,

$$\sigma(q)\equiv\phi(q)+\phi^*(-q),$$

$$\pi(q)\equiv-i[\phi(q)-\phi^*(q)],$$

with the relations of $\sigma^*(q)=\sigma(-q)$ and $\pi^*(q)=\pi(-q)$. These fields are analogous to the σ and π fields in the chiral dynamics, respectively. By using these new fields, we get

$$S_{\rm eff}^{(2)}=\sum_q (\sigma^*(q)D_\sigma^{-1}(q)\sigma(q)+\pi^*(q)D_\pi^{-1}(q)\pi(q)), \tag{23}$$

where two propagators are defined by

$$D_\sigma^{-1}(q)\equiv(X_q+X_q^*+2Y_q)/4,$$

$$D_\pi^{-1}(q)\equiv(X_q+X_q^*-2Y_q)/4.$$

It is evident that the sigma mode represents the amplitude mode of the gap parameter and the pi mode corresponds to the phase mode. In chiral dynamics, the former corresponds to a sigma meson and the latter to a pi meson, respectively [3].

IV. COLLECTIVE MODES

Now we discuss the physical properties of the fluctuation modes or collective motion in detail. Our interest will be put on the fluctuation near the critical temperature. For convenience, our discussions are separated into two cases: one is the collective modes in the Nambu-Goldstone (NG) phase at $T<T_c$ and the other in the Wigner phase at $T>T_c$.

A. $\phi_0\neq 0$ $(T<T_c)$

Each mode corresponds to a field of the diquark with a fractional baryon number. This particle is a boson but similar to the antiquark concerning the color quantum number. The diquark masses should be calculated from zeros of the inverse propagators with $(\vec{q}=0,\omega_q=\nu)$. After the summation of the Matsubara frequency, the propagator for the pi mode is represented by

$$D_\pi^{-1}(\nu)=\frac{8g\beta}{3}\sum_{\vec{p}}{\rm tr}\left[\hat{\varepsilon}^2\frac{(i\nu)^2}{(i\nu)^2-4\hat{E}_p^2}\frac{1-2f(\hat{E}_p)}{2\hat{E}_p}\right], \tag{24}$$

where use is made of the gap equation (26). This equation means that the mass vanishes and the diquark is a Nambu-Goldstone (NG) boson as expected.

In the same way, the inverse propagator for the sigma mode is written as

$$D_\sigma^{-1}(\nu)=\frac{8g\beta}{3}\sum_{\vec{p}}{\rm tr}\left[\hat{\varepsilon}^2\frac{(i\nu)^2-4\hat{\Delta}_0^\dagger\hat{\Delta}_0}{(i\nu)^2-4\hat{E}_p^2}\frac{1-2f(\hat{E}_p)}{2\hat{E}_p}\right]. \tag{25}$$

It is found that the mass of the σ is equivalent to twice the square root of $\hat{\Delta}_0^\dagger\hat{\Delta}_0$. Noting that the effective quark mass is the square root, we have the following mass relation: $M_\pi:M_q:M_\sigma=0:1:2$. This simple relation is called quasi-supersymmetry by Nambu [13]. We have shown that this universal relation is also satisfied in the color superconductivity.

In order to obtain the mass of the sigma mode, we must solve the gap equation, which is read to

$$\frac{8g}{3}\sum_{\vec{p}}{\rm tr}\left[\hat{\varepsilon}^2\frac{1-2f(\hat{E}_p)}{2\hat{E}_p}\right]=1. \tag{26}$$

We have solved this equation numerically and calculated the mass of the sigma mode that is twice the gap energy. The result is shown in Fig. 1. The closed circles represent the mass of the sigma mode, M_σ. When the temperature approaches the critical one (T_c), the mass goes to zero.

As was pointed out below Eq. (16), there are many other fluctuation modes except the present one. It is very important to discuss all the NG bosons appearing in our model [14–20]. Our starting Lagrangian (1) has the following sym-

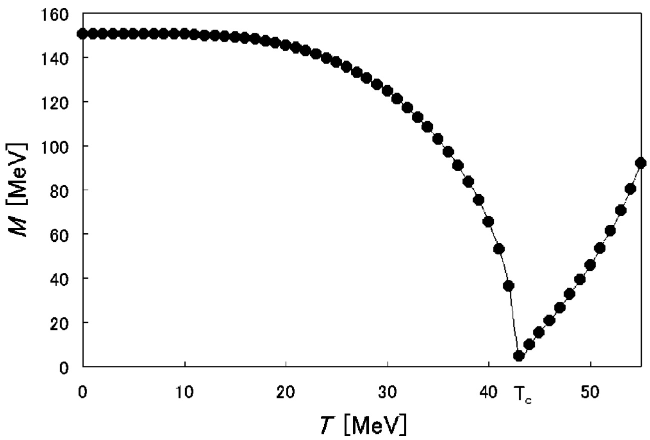

FIG. 1. The mass of the sigma mode as a function of the temperature.

metry: $G=SU(3)_c^L\times SU(3)_c^R\times SU(3)_f^L\times SU(3)_f^R\times U(1)_B\times U(1)_A$. Note that our model has separate color symmetries for L and R quarks different from QCD. These symmetries are broken down to $SU(3)_{c+f}^L\times SU(3)_{c+f}^R$ by the color-flavor locking. According to [21], there are 18 NG bosons in our model. The pi mode discussed in the present paper corresponds to $U(1)_B$ symmetry: $\hat{\Delta}_0\to\hat{\Delta}=\hat{\Delta}_0+\phi\hat{\varepsilon}$ $(q\to e^{i\theta})$.

Moreover, it is instructive to compare our result with that in QCD. The symmetries in QCD are $G=SU(3)_c\times SU(3)_f^L\times SU(3)_f^R\times U(1)_B$, which are broken to $SU(3)_{c+L+R}$. Thus we have 17 NG bosons. In QCD the $U(1)_A$ symmetry is explicitly broken, meaning that the NG boson is only a pseudo-NG boson. However, since the vector color symmetry is gauged in QCD, eight of the 17 NG bosons are eaten, becoming the longitudinal components of eight massive gluons. Thus there are nine NG bosons and one pseudo-NG boson in QCD.

B. $\phi_0=0$ $(T>T_c)$

It is well known that even in the Wigner phase the fluctuation of the order parameter appears near the critical temperature. This fluctuation is called soft mode or precursor [22]. Noting that $X_q^*=X_{-q}$ and $Y_q=0$, the previous action (20) for the scalar field is described by

$$S_{\rm eff}^{(2)}=2\sum_q \phi^\dagger(q)D^{-1}(q)\phi(q), \tag{27}$$

where the inverse propagator is given by

$$\begin{aligned}D^{-1}(q)&\equiv 2X_q\\ &=32g\sum_p \frac{|N_{p,p-q}|^2}{(i\omega-\xi_p)(i\omega-i\nu+\xi_{p-q})}+\beta,\end{aligned} \tag{28}$$

where we have used the relation $\mathrm{tr}\,\hat{\varepsilon}^2=12$. Carrying out the summation of the Matsubara frequencies, we get

$$\begin{aligned}D^{-1}(q)=32g\beta\sum_{\vec{p}}&\frac{f(\xi_{p+1/2q})+f(\xi_{p-1/2q})-1}{\xi_{p+1/2q}+\xi_{p-1/2q}-i\nu}|N_{p,p-q}|^2\\ &+\beta,\end{aligned} \tag{29}$$

where $f(x)\equiv(1+e^{\beta x})^{-1}$ is the Fermi distribution function.

In order to obtain the physical properties of the collective mode, let us transform Eq. (29) from the imaginary time formalism to the real time one. The inverse propagator (29) should be rewritten with the use of the usual analytical continuation: $i\nu\to\omega+i\delta$ where the δ is a positive infinitesimal constant. Then the (inverse) retarded Green's function with $\vec{q}=0$ is given by

$$D_R^{-1}(\omega)=\beta-32g\beta\sum_{\vec{p}}\frac{1-2f(\xi_p)}{2\xi_p-\omega-i\delta}\equiv F(\omega). \tag{30}$$

We have denoted the right-hand side by a new function, $F(\omega)$. Using the well-known formula, it is represented as

$$\begin{aligned}F(\omega)=16g\beta\sum_{\vec{p}}&\left(\frac{\mathcal{P}}{\xi-\dfrac{\omega}{2}}+i\pi\delta\left(\xi-\frac{\omega}{2}\right)\right)\\ &\times[2f(\xi)-1]+\beta\\ &\equiv\mathrm{Re}\,F(\omega)+i\,\mathrm{Im}\,F(\omega).\end{aligned} \tag{31}$$

The mass is defined by a zero point ω such that $F(\omega)=0$. It is, however, important to notice the possibility that the ω has an imaginary part. This is nothing but analogous to the "Landau damping" in plasma physics. If we denote the zero point by $\omega=M-i\Gamma$ and assume the Γ is small, they are determined by the following simultaneous equations [23]:

$$0=\mathrm{Re}\,F(M), \tag{32}$$

$$\Gamma=[\mathrm{Im}\,F(M)]\left(\frac{d}{d\omega}\mathrm{Re}\,F(\omega)\right)^{-1}_{\omega=M}. \tag{33}$$

We can determine the mass of the collective mode from Eq. (32) and the decay width from Eq. (33). The calculated mass is shown in Fig. 1 $(T>T_c)$. The value is zero at critical temperature and increases with the temperature. The calculated decay width is drawn in Fig. 2 together with the corresponding mass. The closed circles represent the mass M as a function of the temperature. On the other hand the closed triangles show the corresponding decay widths. It is seen that both the mass and decay width vanish at the critical temperature T_c and increase gradually as temperature increases. They are the same order of magnitude.

These behaviors are understood analytically as follows. With the use of the gap equation (26), the real part of $F(\omega)$ will be rewritten as

$$\mathrm{Re}\,F(\omega)=32g\beta\sum_{\vec{p}}\left(\frac{2f(\xi_p;T)-1}{2\xi_p-\omega}-\frac{2f(\xi_p;T_c)-1}{2\xi_p-\omega}\right), \tag{34}$$

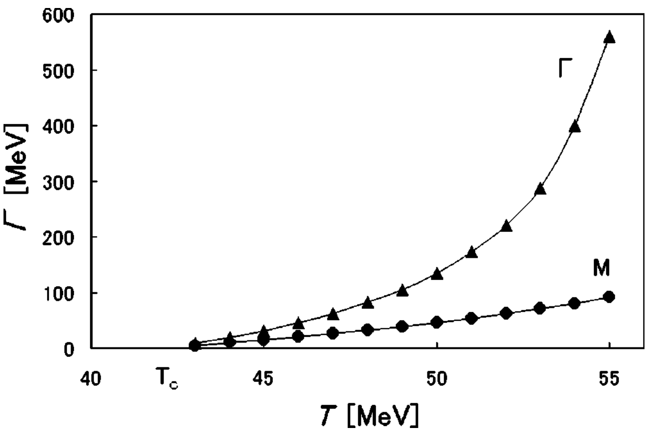

FIG. 2. The mass M and decay width Γ of the collective mode as a function of the temperature.

where $f(x;T)=(1+e^{x/T})^{-1}$ is the Fermi distribution function at temperature T. If the right-hand side is expanded by power series with respect to ω and $(T-T_c)$, it is read to

$$\approx 32 g\beta \sum_{\vec{p}} \left(\frac{2f(\xi_p)}{2\xi_p}(T-T_c)^2 + \frac{2f(\xi_p)-1}{4\xi_p^2}\omega \right). \quad (35)$$

Noting that $\mathrm{Re}\,F(M)=0$, the mass of the collective mode is proportional to $(T-T_c)^2$. It is consistent with the result shown in Fig. 2.

As for the decay width, let us consider the imaginary part of the $F(\omega)$,

$$\mathrm{Im}\,F(\omega)\approx 32\pi g\beta \sum_{\vec{p}} \delta(2\xi_p-\omega) \times \left(\frac{\partial f}{\partial \omega}\omega + \frac{\partial^2 f}{\partial T^2}(T-T_c)^2 \right), \quad (36)$$

where use is made of the same power series expansion. When the ω is equal to M, the imaginary part is proportional to $(T-T_c)^2$. Since the $(d\,\mathrm{Re}\,F)/(d\omega)$ is a constant near $T=T_c$, we see that the decay width is proportional to $(T-T_c)^2$, which is also consistent with the result in Fig. 2.

Several years ago Hatsuda and Kunihiro pointed out that such a soft mode appears in the chiral phase transition. They discuss the spectral function of the system for the signal of the soft mode. In this paper we discuss whether this phenomenon is realized in the color superconductivity. To this end, let us investigate the spectral function of our system just as was done in [2]. This function is derived from the retarded Green's function (30) obtained above. From this equation we get the spectral function,

$$S(\omega)\propto \mathrm{Im}\,D_R(\omega)=\mathrm{Im}\frac{1}{D_R^{-1}(\omega)}. \quad (37)$$

We have calculated the spectral function at various temperature just above the critical one. The numerical results are shown in Fig. 3. It is seen that this function has a sharp peak near the critical temperature. This means that the strong collectivity may be observed from the spectral function. The behavior of the spectral function for low frequency can be understood by the following consideration. First we notice that the imaginary part of the inverse propagator is proportional to ω: $\mathrm{Im}[D^{-1}(\omega)]\propto a\omega$. The proportional constant a is a function of the temperature, $a(T)=a_0/T$. On the other hand, the real part $\mathrm{Re}[D^{-1}(\omega)]$ is represented by $b(T)=b_0(T-T_c)$ because it vanishes at $T=T_c$. Therefore the spectral function is described by $S(\omega)\propto a\omega/(b^2+a^2\omega^2)$. This function has a peak at $\omega=b/a$ and the height is $1/2ab$. As the temperature T approaches to T_c, the position of the peak approaches to zero and the height increases. This enhancement near the critical temperature is observed in Fig. 3.

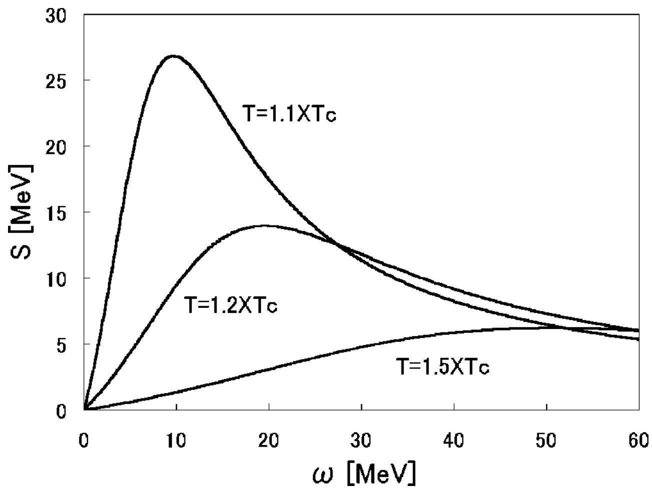

FIG. 3. The spectral function of the system at various temperatures.

Finally let us comment a characteristic phenomenon that this soft mode brings about. It was pointed out that the acceleration of cooling of quark stars is caused by the soft mode on chiral symmetry [4]. Our soft mode is also expected to work in the same way. For example, two high-energy quarks in the hot quark star may be transformed into a soft diquark and cooled with emission of a pair of neutrinos via the following weak interaction process:

$$q_1+q_2\rightarrow q_1'+e^-+\overline{\nu_e}+q_2\rightarrow(q_1'q_2')_{\mathrm{soft}}+\overline{\nu_e}+\nu_e. \quad (38)$$

If Bose-Einstein condensation of the soft mode occurs, a huge number of neutrinos would be created so that very rapid cooling is realized in the quark stars just above the critical temperature. On the other hand, the cooling is strongly suppressed due to the color superconductivity below the same critical temperature as discussed by many authors [24–26]. Such a singular cooling behavior may be a possible sign for the color superconductivity in the hot quark matter.

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